Switching-based Adaptive Output Regulation for Uncertain Systems Affected by a Periodic Disturbance

Guanqi He, Yang Wang, Gilberto Pin, Andrea Serrani and Thomas Parisini

***Abstract*— Disturbance rejection for uncertain systems is a longstanding problem of both theoretical and practical importance. In this paper, a novel switching-based Adaptive Feedforward Controller (AFC) is proposed to remove a priori information on the sign of the plant transfer function at the frequency of interest (the so-called SPR-like condition), which is an undesirable but inevitable requirement for the existing AFC approaches. A distinctive feature of the work presented herein is the improvement of the transient behaviour by virtue of the adoption of a new switching mechanism based on an unnormalized adaptation law. Furthermore, the dimension of the overall controller is kept relatively low regardless of the number of candidate controllers. Boundedness of the trajectory of the closed-loop system and asymptotic zeroing of the output are rigorously proved. The effectiveness of the proposed technique is illustrated by numerical examples.**

I. Introduction

The problem of rejecting periodic disturbance has long been of special interest in control community, due to its commonly appearance in practical applications [1]–[5]. From a methodological perspective, the periodic disturbance cancellation problem can either be cast in the general framework of the output regulation problem [6], [7] or tackled by means of Adaptive Feedforward Cancellation (AFC) techniques [8], [9], for which a large body of literature exists. In the presence of plant model uncertainties, the majority of works in the realm of AFC assumes that the sign of the transfer function of the plant is known a priori and persists over the range of frequencies of interest [10]–[12]. Such hypothesis has been termed as an *SPR-like condition* in the literature [13]. Under the SPR-like condition, AFC-based solutions have been extended to handle the case of unknown frequency of the disturbance [14], [15], as well as to discrete-time systems [16], [17] and nonlinear systems [18]. However, if the crucial information on the frequency response of the plant is absent, conventional AFC strategies like the ones listed above cannot be implemented.

A few works have attempted to remove SPR-like conditions via on-line estimation of the missing information within the AFC framework [19], [20]. However, issues related to asymptotic stability and the problem of non-singularity of the control law were not fully addressed. An alternative approach known as Adaptive Harmonic Steady-state Control (AHSC) was developed in [19], [21], which assume that the plant model is in steady-state and neglect the dynamic interaction between the plant dynamics and the particular AFC algorithm employed. Under the assumption that frequency of disturbance is known, recent works [13] [22] have proposed a multiple-model adaptive control scheme and a state-norm-estimator-based switching strategy, respectively, that indeed remove the necessity of SPR-like conditions. However, these approaches suffer from the explosion of controller dimension or possibly unsatisfactory transient performance.

To address the aforementioned shortcomings of existing AFC methods, this note proposes a novel switching-based strategy to circumvent the need for SPR-like conditions when the frequency of excitation is known. The main idea is inspired by the observation made in [13] that $\mathcal{L}_2$ stability can be achieved without exact convergence of the controller parameters to their true values. We employ the certainty-equivalence controllers proposed in [13] as our base-line candidate controllers and introduce a switching mechanism to identify an 'optimal' controller among a family of candidate controllers, in analogy with the logic-based switching mechanism of [23]–[25]. However, using the classic approach to multiple-model adaptive control based on a bank of parallel observers would dramatically increase the complexity of algorithm even for a single-tone disturbance, let alone a multi-harmonic one. This motivates the main contribution of this work, which lies in the construction of a new switching mechanism characterized by the fact that the switching signal is generated by one second-order adaptive systems. In this way, regardless of the size of the family of candidate controllers, the dimension of the controller remains the same, which is significant and crucial for the further extension to the case of multiple frequencies. Furthermore, we remove the normalization in adaptive law, which speeds-up the identification of the stabilizing controller and the convergence rate of the regulated output. Global stability and convergence are rigorously proved and effectiveness of the algorithm are shown in simulation study.

This paper is organized as follows: Standing assumptions and the formulation of the problem are given in Section II. The design of the candidate controllers and related proofs are presented in Section III. In Section IV, we propose the novel supervisory system with monitor signal and hysteresis switching logic, and prove the convergence of the output. In Section V, we provide illustrative examples to validate the effectiveness of the scheme we proposed scheme.

Guanqi He and Yang Wang are with the School of Information Science and Technology, ShanghaiTech University, Shanghai, China. Gilberto Pin is with Electrolux Italia, S.p.A., Italy. Andrea Serrani is with the Department of Electrical and Computer Engineering, The Ohio State University, Columbus OH – USA. Thomas Parisini is with the Department of Electrical and Electronic Engineering, Imperial College, London UK. This work was supported in part by the Yangfan Program of Shanghai, China, under Grant 21YF1429600. Corresponding author: Y.Wang, wangyang4@shanghaitech.edu.cn

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II. Problem Formulation

Similar to [13], we consider a linear time-invariant single-input single-output (SISO) system described by

$$\begin{aligned}\dot{x}(t) &= A(\mu)x(t) + B(\mu)[u(t) - d(t)], \quad x(0) = x_0 \in \mathbb{R}^n \\ y(t) &= C(\mu)x(t)\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$ and $d \in \mathbb{R}$ represent the state, the input and the output of the plant, and the external disturbance, respectively.

Our target is to design the control $u(t)$ that eliminates the effect of the disturbance $d(t)$ on $y(t)$, under the least information on the plant model (1). More precisely, no information on the structure of $A(\mu)$, $B(\mu)$, $C(\mu)$ is required, apart from the assumption of robust stability given below, and the fact that the plant parameter vector $\mu \in \mathbb{R}^p$ ranges on a given known compact set, $\mathcal{P} \subset \mathbb{R}^p$. System (1) is assumed to be internally stable, robustly with respect to $\mu \in \mathcal{P}$, which is formally stated as follows:

Assumption II.1. There exist constants $a_1, a_2, a_3 \geq 0$ such that the parameterized family $P_x(\mu) : \mathbb{R}^p \to \mathbb{R}^{n\times n}$ of solutions of the Lyapunov equation $P_x(\mu)A(\mu)+A^\top(\mu)P_x(\mu) = -I$ satisfies $a_1 I \leq P_x(\mu) \leq a_2 I, \|P_x(\mu)\| \leq a_3$ for all $\mu \in \mathcal{P}$.

The disturbance $d(t)$ is taken as a sinusoidal signal $d(t) = \psi \cos(\omega^* t + \phi)$ with known frequency $\omega^* > 0$ and unknown amplitude and phase $\psi > 0$, $\phi \in (-\pi, \pi]$. The disturbance is generated by the following exosystem

$$\begin{aligned}\dot{w} &= Sw, \quad w(0) = w_0 \in \mathbb{R}^2 \\ d &= \Gamma w\end{aligned} \tag{2}$$

where $S = \begin{pmatrix} 0 & \omega^* \\ -\omega^* & 0 \end{pmatrix}, \Gamma = \begin{pmatrix} 1 & 0 \end{pmatrix}$, and the uncertainty associated with ψ and ϕ is mapped to the unknown $w_0 \in \mathbb{R}^2$. The problem to be solved is stated as follows:

Problem II.1. Let Assumption II.1 hold for the uncertain plant model (1) affected by a sinusoidal disturbance generated by the exosystem (2). Design a dynamic feedback controller

$$\begin{aligned}\dot{\zeta}_c &= \varphi_c(\zeta_c, y), \quad \zeta_c(0) = \zeta_{c,0} \in \mathbb{R}^m \\ u &= h_c(\zeta_c, y)\end{aligned} \tag{3}$$

such that the trajectory of the closed-loop system originating from arbitrary initial conditions $x_0 \in \mathbb{R}^n$, $w_0 \in \mathbb{R}^2$, $\zeta_{c,0} \in \mathbb{R}^m$ are bounded and the output of the plant satisfies $\lim_{t\to\infty} y(t) = 0$.

Let the parameter vector $\theta \in \mathbb{R}^2$ be defined as

$$\theta^\top(\mu) = \begin{pmatrix} \mathrm{Re}\{W(j\omega^*)\} & -\mathrm{Im}\{W(j\omega^*)\} \end{pmatrix}$$

where $W(s) := C(\mu)(sI - A(\mu))^{-1}B(\mu)$ denotes the transfer function of system (1). Assuming that θ is known, following [13], Problem II.1 can be solved by the interconnection of the *external model of the disturbance*

$$\begin{aligned}\dot{\hat{w}} &= S\hat{w} + Gu_a, \quad \hat{w}(0) = \hat{w}_0 \in \mathbb{R}^2 \\ u &= \Gamma\hat{w}\end{aligned} \tag{4}$$

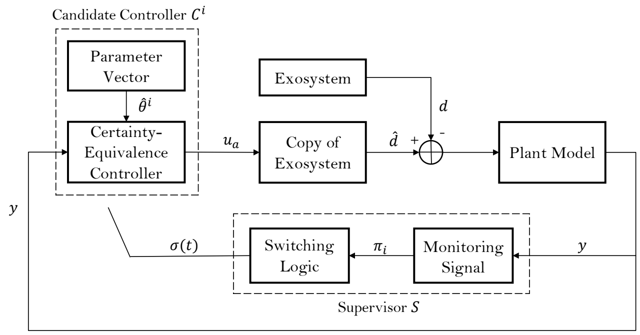

Fig. 1. Block diagram of the proposed switching-based AFC.

with the observer

$$\begin{aligned}\dot{\hat{\zeta}}_o &= \left[S - \varepsilon\theta\theta^\top\right]\hat{\zeta}_o - \alpha G[\Gamma\hat{\zeta}_o - y], \quad \hat{\zeta}_o(0) \in \mathbb{R}^2 \\ u_a &= -\varepsilon\theta^\top\hat{\zeta}_o(t)\end{aligned} \tag{5}$$

provided that the controller gains $\alpha > 0$, $\varepsilon > 0$ are selected to be small enough. Knowledge of θ (or the sign of its components) over the range of frequency of interest constitutes a so-called *SPR-like condition*. Here, the SPR-like condition is replaced by the following weaker requirement, which is much easier to verify in practice:

Assumption II.2. The unknown parameter vector $\theta(\mu)$ satisfies $\theta(\mu) \in \mathrm{int}\ \Theta$ for all $\mu \in \mathcal{P}$, where the compact set Θ is defined as $\Theta := \left\{\theta \in \mathbb{R}^2 | \delta_1^2 \leq \theta_1^2 + \theta_2^2 \leq \delta_2^2\right\}$ for given numbers $0 < \delta_1 < \delta_2$.

Remark II.1. Assumption II.2 is not conservative, as δ_1 and δ_2 can be selected arbitrarily.

Barring further information on θ, the approach of [13] consists in replacing θ in the observer (5) with a suitable estimate $\hat{\theta}$. The main difficulty resides in the selection of the update law for $\hat{\theta}$, which, due to the fact that the parameter set Θ is not convex, requires a cumbersome multi-model estimator. Note that the lower bound δ_1 in Assumption II.2 addresses the issue of non-singularity of the control law, which requires bounding $\hat{\theta}$ away from the origin.

In this paper, we continue the quest for relaxing the SPR-like condition by proposing a novel switching adaptive mechanism for the update law of $\hat{\theta}$. As shown in Fig. 1, the proposed switching control scheme follows the general framework of [26] and mainly consists of two parts. The first one is a family of candidate controllers $\{\mathcal{C}^i\}$, which is designed such that, for any plant model belonging to a family parameterized by the parameter $\mu \in \mathcal{P}$, there exists at least one 'optimal' controller that is capable of stabilizing the closed-loop systems and solving Problem II.1. The selection of such controller will be accomplished by the second part, a high-level supervisor $\mathcal{S}$, which again comprises two subsystems: a monitoring signal that determines which candidate controller should be activated, and a switching logic which is responsible for deciding when to switch.

III. Design of candidate controllers

In this section, we illustrate the design of the family of the candidate controllers and rigorously prove the existence of the 'optimal' controller to reject the disturbance.

To proceed, we first rewrite the interconnection of the plant (1), the exosystem (2) and AFC controller (4) as follows

$$\begin{aligned} \dot z &= A(\mu)z - \Pi(\mu)Gu_a, \quad & z(0) = z_0 \in \mathbb{R}^n \\ \dot\zeta &= S\zeta + Gu_a, & \zeta(0) = \zeta_0 \in \mathbb{R}^2 \\ y &= C(\mu)z + \vartheta^\top(\mu)\zeta \end{aligned} \tag{6}$$

in which we have used the coordinate change $z := x - \Pi(\mu)\zeta$, $\zeta := \hat w - w$, where $\Pi(\mu)$ is the unique solution of the Sylvester equation $\Pi(\mu)S = A(\mu)\Pi(\mu) + B(\mu)\Gamma$. The new parameter vector $\vartheta(\mu) = \begin{pmatrix}\theta_1 & -\theta_2\end{pmatrix}^\top$ is a reparameterization of θ, which also verifies Assumption II.2.

A more convenient representation of the external model of the disturbance in (6) is obtained making use of the change of coordinate $\zeta_o = M_o^{-1}\zeta$,where

$$M_o = \frac{1}{\|\vartheta\|^2}\begin{pmatrix}\vartheta_1 & -\vartheta_2 \\ \vartheta_2 & \vartheta_1\end{pmatrix}.$$

In the new coordinates (z, ζ_o) the interconnection of the plant and the external model reads as

$$\begin{aligned} \dot z &= A(\mu)z - \Pi(\mu)Gu_a \\ \dot\zeta_o &= S\zeta_o + \theta u_a, \quad \zeta_o(0) = \zeta_{o,0} \in \mathbb{R}^2 \\ y &= C(\mu)z + \Gamma\zeta_o \end{aligned} \tag{7}$$

For system (7), Problem II.1 is recast as follows:

Problem III.1. Let Assumption II.1 and II.2 hold. Find a stabilizing control law u_a such that the trajectories of the closed-loop system (7) originating from any initial conditions $z_0 \in \mathbb{R}^n$, $\zeta_{o,0} \in \mathbb{R}^2$ are bounded and the output of the plant satisfies

$$\lim_{t\to\infty} y(t) = 0.$$

In this work, we adopt a similar form for u_a as in [13], but instead of using a time-varying estimate $\hat\theta(t)$, we introduce $N \in \mathbb{N}$ candidate controllers $\{\mathcal{C}^i\}$ as follows

$$u_a^i(t) = -\varepsilon\hat\theta_i^\top\hat\zeta_o(t), \quad i \in \mathcal{I} := \{1, 2, \cdots, N\} \tag{8}$$

where $\hat\theta_i \in \Theta$ is a constant estimate of θ satisfying $\hat\theta_i \neq \hat\theta_j$ for $i \neq j$. The signal $\hat\zeta_o(t)$ is generated by the observer

$$\dot{\hat\zeta}_o = S\hat\zeta_o + \hat\theta_\sigma u_a(t) - \alpha G[\Gamma\hat\zeta_o - y], \quad \hat\zeta_o(0) \in \mathbb{R}^2 \tag{9}$$

where $\sigma \in \mathcal{I}$ denotes the index[1] of the candidate controller that is currently active. Then, the control signal u_a is naturally chosen as

$$u_a(t) = u_a^\sigma(t) \tag{10}$$

Remark III.1. A remarkable feature of the candidate controller (8) is that the dynamic of the observer (9) is shared among all candidate controllers. Therefore, the complexity of the algorithm does not grow with the number N of controllers $\{\mathcal{C}^i\}$.

[1]The selection of the candidate controller is determined by the supervisory system, which will be specified in Section IV.

The next result is instrumental to determine the requirements for the design of the candidate controllers, in terms of the selection of the estimates $\hat\theta_i$, $i \in \mathcal{I}$, that ensures the existence of at least one candidate controller that is able to solve Problem III.1.

Proposition III.1. Let Assumption II.1 and II.2 hold. Fix $\sigma \in \mathcal{I}$. Then, there exist constants $\varepsilon^* > 0$, $\alpha^* > 0$ and $\delta^* > 0$, all independent on the controller gains, such that for any $\varepsilon \in (0, \varepsilon^*)$ and any $\alpha \in (0, \varepsilon\alpha^*)$ the fixed controller (9)-(10) solves Problem III.1 if

$$||\hat\theta_\sigma - \theta|| \le \varepsilon\delta^* =: \bar\delta \tag{11}$$

Proof. We prove this Lemma by showing that, if the inequality (11) is verified for the active controller, the closed-loop system is asymptotically stable. Substituting the control law (10) into the observer (9), one obtains

$$\dot{\hat\zeta}_o = \left[S - \varepsilon\hat\theta_\sigma(t)\hat\theta_\sigma^\top(t)\right]\hat\zeta_o - \alpha G[\Gamma\hat\zeta_o - y] \tag{12}$$

Next, define the observation error $\tilde\zeta_o := \hat\zeta_o - \zeta_o$ whose dynamics read as

$$\begin{aligned} \dot{\tilde\zeta}_o &= F_\alpha\tilde\zeta_o + \tilde\theta_\sigma u_a + \alpha GC(\mu)z \\ \tilde y &= \Gamma\tilde\zeta_o - C(\mu)z \end{aligned} \tag{13}$$

where $\tilde y = y - \Gamma\hat\zeta_o$, $\tilde\theta_\sigma := \hat\theta_\sigma - \theta$ and $F_\alpha := S - \alpha G\Gamma$. Note that F_α is Hurwitz for all $\alpha > 0$, since its characteristic polynomial $p(s) = s^2 + \alpha s + (w^*)^2$. To proceed further with the analysis, we follow a similar procedure as in [13] by expressing $\tilde y(t)$ as $\tilde y(t) = \tilde y_1(t) + \tilde y_2(t)$, where the signals

$$\begin{aligned} \tilde y_1(t) &= \Gamma\int_0^\top e^{F_\alpha(t-\tau)}\tilde\theta_\sigma(\tau)u_a(\tau)d\tau \\ \tilde y_2(t) &= \alpha\Gamma\int_0^\top e^{F_\alpha(t-\tau)}GC(\mu)z(\tau)d\tau - C(\mu)z(t) \end{aligned}$$

admit LTI realizations

$$\dot\xi_1 = F_\alpha^\top\xi_1 + Gu_a, \qquad \tilde y_1 = \tilde\theta_\sigma^\top\xi_1 \tag{14}$$

$$\dot\xi_2 = F_\alpha^\top\xi_2 + \alpha GC(\mu)z, \qquad \tilde y_2 = \Gamma\xi_2 - C(\mu)z \tag{15}$$

respectively. This yields the following non-minimal realization for the closed-loop system (7), (9) and (13)

$$\begin{aligned} \dot{\hat\zeta}_o &= F_\varepsilon\hat\zeta_o - \alpha G(\tilde\theta_\sigma^\top\xi_1 + \Gamma\xi_2 - C(\mu)z) \\ \dot\xi_1 &= F_\alpha^\top\xi_1 - \varepsilon G\hat\theta_\sigma^\top\hat\zeta_o \\ \dot\xi_2 &= F_\alpha^\top\xi_2 + \alpha GC(\mu)z \\ \dot z &= A(\mu)z + \varepsilon\Pi(\mu)G\hat\theta_\sigma^\top\hat\zeta_o \\ y &= \Gamma\hat\zeta_o - \tilde\theta_\sigma^\top\xi_1 - \Gamma\xi_2 + C(\mu)z \end{aligned} \tag{16}$$

where $F_\varepsilon := S - \varepsilon\hat\theta_\sigma\hat\theta_\sigma^\top$ is Hurwitz for all positive ε and non-zero $\hat\theta_\sigma$. For the closed-loop system (16), consider the following Lyapunov candidate function

$$V_\alpha(t) = V_0(t) + \alpha V_1(t) + V_2(t)$$

where

$$V_0 = \hat{\zeta}_o^\top P_\varepsilon \hat{\zeta}_o \,,\; V_1 = \xi_1^\top P_\alpha \xi_1 \,,\; V_2 = a z^\top P_x z + \xi_2^\top P_\alpha \xi_2$$

with $a > 0$ a constant to be determined, and P_ε, P_α and P_x are positive definite matrices that satisfy the properties listed in Assumption II.1, and Property III.1, Property III.2 below:

Property III.1. There exists a scalar $k_1 > 0$, and constants $c_3^\varepsilon \geq c_2^\varepsilon > c_1^\varepsilon > 0$ such that the solution $P_\varepsilon : (\varepsilon, \hat{\theta}_\sigma) \mapsto \mathbb{R}^{2\times 2}$ of the parameterized family of Lyapunov equations

$$P_\varepsilon F_\varepsilon + F_\varepsilon^\top P_\varepsilon = -\varepsilon \hat{\theta}_\sigma^\top \hat{\theta}_\sigma I$$

satisfies $c_1^\varepsilon I \leq P_\varepsilon \leq c_2^\varepsilon I$ and $\|P_\varepsilon\| \leq c_3^\varepsilon$ for all $(\varepsilon, \hat{\theta}_\sigma) \in (0, k_1] \times \Theta$. Note that c_3^ε can be determined independently of $\varepsilon \in (0, k_1]$.

Property III.2. There exists a scalar $k_2 > 0$, and constants $c_3^\alpha \geq c_2^\alpha > c_1^\alpha > 0$ such that the solution $P_\alpha : \alpha \mapsto \mathbb{R}^{2\times 2}$ of the parameterized family of Lyapunov equations

$$P_\alpha F_\alpha + F_\alpha^\top P_\alpha = -\alpha I$$

satisfies $c_1^\alpha I \leq P_\alpha \leq c_2^\alpha I$ and $\|P_\alpha\| \leq c_3^\alpha$ for all $\alpha \in (0, k_2]$. Note that c_3^α can be determined independently of $\alpha \in (0, k_2]$.

The proofs of these two properties are omitted here due to space limitation of space, and can be found in [13]. Evaluation of the derivative of V_0 along the trajectory of system (16) yields

$$\begin{aligned}\dot{V}_0 &= -\varepsilon \|\hat{\theta}_\sigma\|^2 \|\hat{\zeta}_o\|^2 - 2\hat{\zeta}_o^\top P_\varepsilon \alpha G(\tilde{\theta}_\sigma^\top \xi_1 + \Gamma \xi_2 - C(\mu) z) \\ &\leq -\frac{\varepsilon \delta_1^2}{2}\|\hat{\zeta}_o\|^2 + \frac{6\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2}\left(\|\tilde{\theta}_\sigma^\top \xi_1\|^2 + \|\xi_2\|^2 + \rho_c^2 \|z\|^2\right) \quad (17)\end{aligned}$$

where Young's inequality has been applied to the cross terms, the symbol $\rho_{(\cdot)} := \max_{\mu \in \mathcal{P}} \|(\cdot)(\mu)\|$ denotes the maximum value of the norm of the corresponding μ-dependent matrix. Similarly, the Lie derivatives of V_1 and V_2 read as

$$\begin{aligned}\dot{V}_1 &= -\alpha \|\xi_1\|^2 - 2\xi_1^\top P_\alpha \varepsilon G \hat{\theta}_\sigma^\top \hat{\zeta}_o \\ &\leq -\frac{\alpha}{2}\|\xi_1\|^2 + \frac{2\varepsilon^2 (c_3^\alpha)^2 \delta_2^2}{\alpha}\|\hat{\zeta}_o\|^2, \\ \dot{V}_2 &= -a\|z\|^2 + 2a z^\top P_x \varepsilon \Pi G \hat{\theta}_\sigma \hat{\zeta}_o - \alpha \|\xi_2\|^2 + 2\xi_2^\top P_\alpha \alpha G C z \\ &\leq -2\alpha (c_3^\alpha)^2 \rho_c^2 \|z\|^2 - \frac{\alpha}{2}\|\xi_2\|^2 + 16\alpha \varepsilon^2 \rho_2^2 \delta_2^2 \|\hat{\zeta}_o\|^2, \qquad (18)\end{aligned}$$

where a has been set to $a = 8\alpha (c_3^\alpha)^2 \rho_c^2$ and $\rho_2^2 := a_3^2 (c_3^\xi)^2 \rho_c^2 \rho_\Pi^2$. Combining (17) and (18) one obtains

$$\begin{aligned}\dot{V}_\alpha \leq & -\left(\frac{\varepsilon \delta_1^2}{2} - 2\varepsilon^2 (c_3^\alpha)^2 \delta_2^2 - 16\alpha \varepsilon^2 \rho_2^2 \delta_2^2\right)\|\hat{\zeta}_o\|^2 \\ & -\left(\frac{\alpha^2}{2} - \frac{6\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2}\|\tilde{\theta}_\sigma\|^2\right)\|\xi_1\|^2 - \left(\frac{\alpha}{2} - \frac{6\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2}\right)\|\xi_2\|^2 \\ & -\left(2\alpha (c_3^\alpha)^2 \rho_c^2 - \frac{6\alpha^2 (c_3^\varepsilon)^2 \rho_c^2}{\varepsilon \delta_1^2}\right)\|z\|^2. \qquad (19)\end{aligned}$$

After tedious calculation, one obtains that the selection

$$\varepsilon^* := \min\left\{k_1, \frac{\delta_1^2}{4\delta_2^2[(c_3^\alpha)^2 + 8k_2\rho_2^2]}\right\} \qquad (20)$$

$$\alpha^* := \min\left\{\frac{\alpha_0}{3}, \frac{\alpha_1}{12}\right\}, \quad \delta^* := \frac{\alpha_1}{12} \qquad (21)$$

where

$$\alpha_0 = \frac{(c_3^\alpha)^2 \delta_1^2}{(c_3^\varepsilon)^2}, \qquad \alpha_1 = \frac{\delta_1^2}{(c_3^\varepsilon)^2}$$

is such that, for any fixed $\varepsilon \in (0, \varepsilon^*)$, if the observer gain satisfies $\alpha \in (0, \alpha^* \varepsilon)$ and the candidate controller satisfies (11), then $\dot{V}(t) \leq -\rho^* V(t)$ for some positive constant ρ^*, which implies exponential stability of the closed-loop system. □

With the result of Proposition III.1 in mind, once $\varepsilon \in (0, \varepsilon^*)$ has been fixed, the family of candidate controllers shall be built to ensure that there exists a non-empty subset $\mathcal{J} \subset \mathcal{I}$ such that $\|\hat{\theta}_j - \theta\| \leq \varepsilon \delta^*$ for all $j \in \mathcal{J}$.

IV. Design of Supervisory System

Following the results of the previous section, the issue is how to select the 'optimal' candidate controller that is capable of solving Problem III.1. This goal will be accomplished by a high-level supervisor system $\mathcal{S}$ that comprises a novel monitoring signal generator and a hysteresis switching logic.

A. Monitoring Signals π_i

Let $\pi_i(\cdot)$ denote the signal that serves as the performance index of the corresponding candidate controller, $\mathcal{C}_i$. Differently from the majority of switching-based methods [26]–[28], here we use the distance between a time-varying estimate of the unknown parameter θ, denoted as $\eta(t)$, and the constant parameter $\hat{\theta}_i$ adopted by the candidate controller, namely

$$\pi_i(t) := \|\eta(t) - \hat{\theta}_i\|, \quad i \in \mathcal{I}$$

as the monitoring signals. The logic behind this choice is quite straightforward, as $\eta(t)$ approaches a neighborhood of θ, the candidate controller with the smallest $\pi_i(t)$ is likely to be the 'optimal' one. A remarkable feature of the proposed monitoring signal is that, given $\eta(t)$, the calculation of π_i requires no extra dynamics, hence, the complexity of the algorithm is not influenced by the size of the family of candidate controllers.

The output equation of system (16), prompts the following selection of the update law for the estimate $\eta(t)$

$$\begin{aligned}\dot{\eta}(t) &= -\gamma \hat{\xi}_1 (\Gamma \hat{\zeta}_o - y - \tilde{\theta}_\eta^\top \hat{\xi}_1), \quad \eta(0) = \hat{\theta}_{\sigma(0)} \\ &= -\gamma \hat{\xi}_1 \big((\hat{\theta}_\sigma - \hat{\theta}_\eta)^\top \hat{\xi}_1 + \Gamma \xi_2 - C(\mu) z\big) \\ &= -\gamma \hat{\xi}_1 (\tilde{\eta}_\theta^\top \hat{\xi}_1 + \Gamma \xi_2 - C(\mu) z) =: \varphi \qquad (22)\end{aligned}$$

where $\gamma > 0$ is a tuning gain, $\tilde{\theta}_\eta(t) = \hat{\theta}_{\sigma(t)} - \eta(t)$, $\tilde{\eta}_\theta(t) = \eta(t) - \theta$ and $\sigma \in \mathcal{I}$ denotes the index of candidate controller that is currently activated. Note that we have replaced ξ_1 with the estimate $\hat{\xi}_1$ provided by the open loop observer

$$\dot{\hat{\xi}}_1 = F_\alpha^\top \hat{\xi}_1 + G u_a, \quad \hat{\xi}_1(0) = 0 \in \mathbb{R}^2 \qquad (23)$$

Since $F_\alpha^\top$ is Hurwitz, one can easily conclude that $\hat{\xi}_1$ converges to ξ_1 exponentially fast.

Let the time sequence $\{t_k\}, k = 0, 1, 2, \cdots$ denotes the time instants at which switching occurs, and assume without loss of generality that $t_0 = 0$. The next result illustrates that, between any two switching, if $\hat{\theta}_\sigma$ is sufficiently close to $\eta(t)$, then the active controller would solve Problem III.1.

Proposition IV.1. Let $[t_k, t_{k+1})$ be the interval of time between two consecutive switches. Consider the closed-loop system (16) along with the adaptive law (22). If the gains ε,α are sufficiently small and, for all $t \in [t_k, t_{k+1})$,

$$\|\hat{\theta}_{\sigma(t)} - \eta(t)\| < \bar{\mu} \tag{24}$$

where $\bar{\mu} > 0$ is a constant given in (27) below, then the active controller $u_a^\sigma(t)$ solves Problem III.1 for $t_{k+1} = +\infty$.

Remark IV.1. The proposition implies that, if the gains are suitably chosen and condition (24) holds, then the active controller is an 'optimal' one in the given interval.

Proof. Consider the Lyapunov candidate function $V_\beta = V_\alpha + bV_\eta$ where $b = \frac{42\alpha^2(c_3^\xi)^2}{\varepsilon\delta_1^2}$ and $V_\eta = \frac{1}{2\gamma}\tilde{\eta}_\theta^\top\tilde{\eta}_\theta$. Then, with the aid of the update law (22), one obtains

$$\begin{aligned}\dot{V}_\eta =& -\tilde{\eta}_\theta^\top\hat{\xi}_1[\tilde{\eta}_\theta^\top\hat{\xi}_1 + \Gamma\xi_2 - C(\mu)z] \\ \leq& -\frac{1}{2}\|\tilde{\eta}_\theta^\top\xi_1\|^2 + \|\xi_2\|^2 + \rho_c^2\|z\|^2\end{aligned} \tag{25}$$

where Young's inequality has been use to handle the cross terms. Combining the derivative of V_α and V_η, and applying Young's inequality again, one obtains an inequality similar to (19) as follows:

$$\begin{aligned}\dot{V}_\beta \leq& -\left(\frac{\varepsilon\delta_1^2}{2} - 2\varepsilon^2(c_3^\alpha)^2\delta_2^2 - 16\alpha\varepsilon^2\rho_2^2\delta_2^2\right)\|\hat{\zeta}_o\|^2 \\ &-\left(\frac{\alpha^2}{2} - \frac{8\alpha^2(c_3^\xi)^2}{\varepsilon\delta_1^2}\|\tilde{\theta}_\eta\|^2\right)\|\xi_1\|^2 - \left(\frac{21\alpha^2(c_3^\xi)^2}{\varepsilon\delta_1^2}\right)\|\tilde{\eta}_\theta^\top\xi_1\|^2 \\ &-\left(\frac{\alpha}{2} - \frac{8\alpha^2(c_3^\xi)^2}{\varepsilon\delta_1^2} - \frac{42\alpha^2(c_3^\xi)^2}{\varepsilon\delta_1^2}\right)\|\xi_2\|^2 \\ &-\left(2\alpha(c_3^\alpha)^2\rho_c^2 - \frac{8\alpha^2(c_3^\xi)^2}{\varepsilon\delta_1^2} - \frac{42\alpha^2(c_3^\xi)^2\rho_c^2}{\varepsilon\delta_1^2}\right)\|z\|^2\end{aligned} \tag{26}$$

It is worth noting that the main difference between (19) and (26) lies in the coefficient of $\|\xi_1\|^2$, where $\|\tilde{\theta}_\sigma\|$ is replaced with $\|\tilde{\theta}_\eta\|$. Consequently, negattive semi-definiteness of $\dot{V}_\beta$ can be ensured by the condition (24) and suitable selections of α, ε and $\bar{\mu}$ of the form

$$\varepsilon \in (0, \varepsilon^*), \quad \alpha \in (0, \alpha'\varepsilon), \quad \bar{\mu} \in (0, \mu'\varepsilon)$$

with ε^* is given by (20) and

$$\alpha' := \min\left\{\frac{\alpha_0}{25}, \frac{\alpha_1}{100}\right\}, \quad \mu' := \frac{\alpha_1}{16}. \tag{27}$$

Referring to (16), it can be readily verified that all signals of the closed-loop system are uniformly bounded for $t \in [t_k, t_{k+1})$. Moreover, by (26), it follows that $\int_{t_k}^{t_{k+1}} y^2(\tau)\mathrm{d}\tau$ exists and is finite. Assume $t_{k+1} = +\infty$, then it follows from Barbalat's lemma that $y(t)$ converges to the origin. □

B. Switching Logic

From the previous result, one can readily draw the conclusion that if the family of the candidate controller is designed such that the condition (11) is verified with $\bar{\delta} \leq \bar{\mu}$ and $\eta(t) \in \Theta$, i.e., $\delta_1 \leq ||\eta(t)|| \leq \delta_2$, then in view of Propositions III.1 and IV.1, there must be at least one candidate controller satisfying (24). If there is more than one

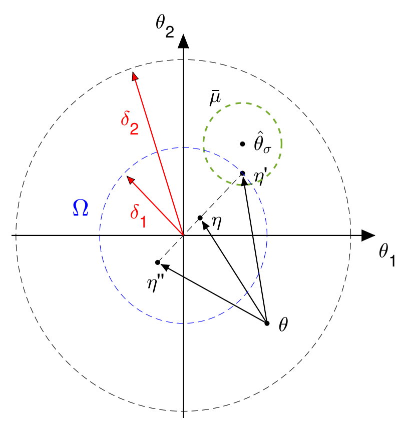

Fig. 2. Sketch of the relation of parameter vectors in (31) in the case of $\|\eta(t)\| < \delta_1$. The vector η'' is on the line connecting η with η' and satisfies $\|\eta'' - \eta\| = \|\eta - \eta'\|$. Also, $\tilde{\eta}_\theta + \tilde{\eta} = \eta' - \theta$ and $\tilde{\eta}_\theta - \tilde{\eta} = \eta'' - \theta$.

stabilizing candidate, to avoid false switching, controllers are selected via the hysteresis switching logic [26]

$$\sigma(t) = \arg\min_{\sigma(t^-),j}\left\{\pi_j, \pi_{\sigma(t^-)} - h\right\} \quad, j \in \mathcal{I} - \sigma(t^-) \tag{28}$$

where $h \in (0, \frac{\bar{\mu}}{2})$ is a positive design parameter that avoids infinite fast switching.

Remark IV.2. The main difference setting the proposed switching mechanism apart from classical techniques (see, for example, [27], [29] is that the supervisory system does not rely on a group of observers running in parallel to identify the 'optimal' candidate. Therefore, the complexity of the overall switching control architecture is greatly reduced.

The last but critical step is to confine the estimate $\eta(t)$ to the parameter set Θ. Clearly, this cannot be guaranteed with the adaptive law (22). Considering the non-convex shape of the parameter set Θ, there are two scenarios that need to be taken care of separately.

First, the upper norm-bound can be achieved by adding a standard projection operator [30] at the boundary as follows:

$$\dot{\eta} = \begin{cases} \varphi & \text{if} \|\eta\| < \delta_2 \\ & \text{or if } \|\eta\| = \delta_2 \text{ and } \varphi^\top\eta \leq 0 \\ \left(I - \frac{\eta\eta^\top}{\|\eta\|^2}\right)\varphi & \text{otherwise} \end{cases} \tag{29}$$

Referring to [30, Chapter 4.4], it can be easily proved that the projection modification will retain all the features of the original update law (22).

For the lower bound δ_1, the solution is non-trivial. Note that, since $\eta(t)$ is not directly employed in the control signal, but merely serves as a monitoring signal, it is not essential to bound it away from the origin (Note that, the family of candidate controller still needs to be designed to verify $||\hat{\theta}^i|| \in \Theta$). However, if $\eta(t)$ enters the inner circle of Θ, i.e., $\|\eta\| < \delta_1$, the critical condition (24) in Lemma IV.1 may be violated, as in view of (27), δ_1 is in general much larger than $\bar{\mu}$. To solve this problem, we modify the monitoring signal as follows

$$\pi_i(t) := \begin{cases} \|\hat{\theta}_i - \eta(t)\| & \text{if } ||\eta|| \geq \delta_1 \\ \|\hat{\theta}_i - \eta'(t)\| & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{I}, \quad (30)$$

where $\eta'(t)$ denotes the projection of $\eta(t)$ on Ω along its radial direction, and Ω is the inner boundary circle of set Θ (see Figure 2).

Defining $\tilde{\theta}^i_{\eta'} = \hat{\theta}_i - \eta'$, $\tilde{\eta} = \eta' - \eta$ for all $i \in \mathcal{I}$, one can rewrite the parameter error $\tilde{\theta}_i$ as

$$\tilde{\theta}_i = \tilde{\theta}^i_{\eta'} + \tilde{\eta} + \tilde{\eta}_\theta. \qquad (31)$$

Suppose the family of candidate controller is properly designed; then, for any $\eta'(t)$, one can find a non-empty set $\mathcal{J}$ such that $\|\hat{\theta}_j - \eta'\| \leq \bar{\mu}, \quad \forall j \in \mathcal{J}$. Next, with the aid of the auxiliary variable $\eta'(t)$, we prove the boundedness and convergence properties of closed-loop trajectories when $\|\eta(t)\| < \delta_1$:

Proposition IV.2. Suppose $\|\eta(t)\| < \delta_1$ for the time interval $t \in [t_k, t_{k+1})$. Consider the closed-loop system (16) where the active controller $u^\sigma_a(t)$ is selected by the supervisory system consisting of (28) and (30) along with the adaptive law (29). All signals remain uniformly bounded and the truncated $\mathcal{L}_2$-norm of output is finite if the gain parameters ε and α are chosen sufficiently small and inequality

$$\pi_\sigma(t) := \|\tilde{\theta}^\sigma_{\eta'}\| < \bar{\mu}^* \qquad (32)$$

is verified with $\bar{\mu}^* > 0$ given by (36).

Proof. Given the auxiliary signal η', the dynamics of the observer $\hat{\zeta}_o$ in (16) can be rewritten as

$$\dot{\hat{\zeta}}_o = F_\varepsilon \hat{\zeta}_o - \alpha G[(\tilde{\theta}_{\eta'} + \tilde{\eta} + \tilde{\eta}_\theta)^\top \xi_1 + \Gamma \xi_2 - C(\mu) z] \quad (33)$$

Consider the same candidate Lyapunov function V_β and compute its derivative along the trajectory of the closed-loop system to obtain

$$\begin{aligned} \dot{V}_\beta \leq &- \left(\frac{\varepsilon \delta_1^2}{2} - 2\varepsilon^2 (c_3^\alpha)^2 \delta_2^2 - 16\alpha\varepsilon^2 \rho_2^2 \delta_2^2 \right) \|\hat{\zeta}_o\|^2 \\ &- \left(\frac{\alpha^2}{2} - \frac{10\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2} \|\tilde{\theta}^\sigma_{\eta'}\|^2 \right) \|\xi_1\|^2 - \left(\frac{\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2} \right) \|\tilde{\eta}_\theta^\top \xi_1\|^2 \\ &- \left(\frac{\alpha}{2} - \frac{10\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2} - \frac{42\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2} \right) \|\xi_2\|^2 \\ &- \left(2\alpha (c_3^\alpha)^2 \rho_c^2 - \frac{10\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2} - \frac{42\alpha^2 (c_3^\varepsilon)^2 \rho_c^2}{\varepsilon \delta_1^2} \right) \|z\|^2 \\ &- \left(\frac{10\alpha^2 (c_3^\varepsilon)^2}{\varepsilon \delta_1^2} \right) (\|\tilde{\eta}_\theta^\top \xi_1\|^2 - \|\tilde{\eta}^\top \xi_1\|^2) \end{aligned} \qquad (34)$$

Compared with (19) and (26), we notice that the error signal in the coefficient of $\|\xi_1\|^2$ now becomes $\|\tilde{\theta}^\sigma_{\eta'}\|^2$, which is assumed to be less than $\bar{\mu}^*$ by virtue of (32). The new term $\Delta_\eta := \|\tilde{\eta}_\theta^\top \xi_1\|^2 - \|\tilde{\eta}^\top \xi_1\|^2$ appearing in the final line of (34) can be expressed as

$$\begin{aligned} \|\tilde{\eta}_\theta^\top \xi_1\|^2 - \|\tilde{\eta}^\top \xi_1\|^2 &= (\tilde{\eta}_\theta^\top \xi_1 + \tilde{\eta}^\top \xi_1)^\top (\tilde{\eta}_\theta^\top \xi_1 - \tilde{\eta}^\top \xi_1) \\ &= \xi_1^\top P_\eta \xi_1 \end{aligned} \qquad (35)$$

where $P_\eta = (\tilde{\eta}_\theta + \tilde{\eta})(\tilde{\eta}_\theta - \tilde{\eta})^\top \in \mathbb{R}^{2\times 2}$. It can be verified that one eigenvalue of P_η is always zero, while the other one is $\lambda_2 := (\tilde{\eta}_\theta + \tilde{\eta})^\top (\tilde{\eta}_\theta - \tilde{\eta})$. Thus, the new term Δ_η is positive semi-definite as long as $\lambda_2 \geq 0$. Since we have $\|\tilde{\eta}\| < \delta_1 \leq \|\theta\|$, as shown in Fig. 2, one can see that the vector $\eta' - \eta''$ crosses the origin but with Euclidean norm not greater than the diameter of Ω. Therefore, it follows from basic planar geometry that $\langle \tilde{\eta}_\theta + \tilde{\eta}, \tilde{\eta}_\theta - \tilde{\eta} \rangle \leq \pi/2$ and $\lambda_2 \geq 0$. Consequently, P_η is positive semi-definite, thus Δ_η is non-negative.

Finally, to ensure $\dot{V}_\beta \leq 0$, we again need to select α, ε and $\bar{\mu}^*$ to satisfy the restrictions

$$\varepsilon \in (0, \varepsilon^*), \quad \alpha \in (0, \alpha''\varepsilon), \quad \bar{\mu}^* \in (0, \mu''\varepsilon) \qquad (36)$$

where ε^* is given by (20) and $\alpha'' := \min\left\{\frac{\alpha_0}{26}, \frac{\alpha_1}{104}\right\}, \quad \mu'' = \frac{\alpha_1}{20}$. The proof for uniformly boundedness of all signals and finiteness of the truncated $\mathcal{L}_2$-norm from t_k to t_{k+1} follows similar arguments, and thus are omitted. □

Finally, the main result is summarized in next theorem.

Theorem IV.1. Suppose that Assumption II.1 and II.2 hold, the disturbance rejection problem II.1 is solved by a switching-based AFC scheme consisting of the controller (10), the switching logic (28) and the monitoring signals (30) along with the adaptive law (29), *if:*

i) The family of candidate controllers is designed such that the condition (11) is satisfied with

$$\bar{\delta} \in (0, \bar{\mu}^*) \qquad (37)$$

ii) The gains α, ε and $\bar{\mu}^*$ are chosen to verify (36).

Proof. Consider the same candidate Lyapunov function V_β. For each $t \in [t_k, t_{k+1})$, $k = 0, 1, 2, \cdots$, if the condition i) and ii) stated above are satisfied, then, referring to Propositions III.1, IV.1 and IV.2, it can be easily verified that the time derivative of V_β along the solution of (16) and estimator (29) satisfies

$$\dot{V}_\beta \leq - r_0 \left(\|\hat{\zeta}_o\|^2 + \|\xi_1\|^2 + \|\xi_2\|^2 + \|z\|^2 + \|\tilde{\eta}_\theta^\top \xi_1\|^2 \right) \quad (38)$$

for some positive constant r_0 that depends on tuning parameters. Notice that, (38) holds regardless of the value of estimate $\eta(t)$. Moreover, all signals involved in V_β are continuous, hence $V_\beta(t_k-) = V_\beta(t_k)$. This shows boundedness of all variables of the closed-loop system (16) for all $t \geq 0$. To show that the output y is regulated to zero, recall the form of y in (16):

$$y = \Gamma \hat{\zeta}_o - \tilde{\theta}_\eta^\top \xi_1 - \tilde{\eta}_\theta^\top \xi_1 - \Gamma \xi_2 + C(\mu) z$$

Bearing in mind that $\|\tilde{\theta}_\eta\| \leq \bar{\mu}$ all the time and Γ, $C(\mu)$ are constant matrices, one obtains

$$\begin{aligned} &\int_0^\infty \|y(\tau)\|^2 \mathrm{d}\tau = \sum_{i=0}^\infty \int_{t_i}^{t_{i+1}} \|y(\tau)\|^2 \mathrm{d}\tau \\ &\leq \sum_{i=0}^\infty \int_{t_i}^{t_{i+1}} r_1 \left(\|\hat{\zeta}_o\|^2 + \|\xi_1\|^2 + \|\xi_2\|^2 + \|z\|^2 + \|\tilde{\eta}_\theta^\top \xi_1\|^2 \right) \mathrm{d}\tau \end{aligned}$$

for some constant $r_1 > 0$. In view of (38), it follows that

$$\int_0^\infty \|y(\tau)\|^2 \mathrm{d}\tau \leq \sum_{i=0}^{\infty} \int_{t_i}^{t_{i+1}} -\frac{r_1}{r_0}\dot{V}_\beta \mathrm{d}\tau$$
$$\leq \frac{r_1}{r_0}\sum_{i=0}^{\infty}(V_\beta(t_i) - V_\beta(t_{i+1})) \leq \frac{r_1}{r_0}V_\beta(0),$$

which implies $y \in \mathcal{L}_2$. Then referring to (7), (8) and (10), since $\hat{\theta}_\sigma$, $\hat{\zeta}_o$, z and y are bounded and piece-wise continuous, it holds that u_a and ζ_o are bounded. Consequently, both $\dot{\zeta}_o$ and $\dot{z}$ are bounded, hence $\dot{y} \in \mathcal{L}_\infty$. Since $y \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\dot{y} \in \mathcal{L}_\infty$ by Barbalat's Lemma, it can be concluded that $y(t) \to 0$ as $t \to 0$. This completes the proof. □

Remark IV.3. In Theorem IV.1, we prove the convergence of the output no matter whether the switching stop or not. As a matter of fact, from (38), it can be concluded that $\tilde{\eta}_\theta^\top \tilde{\eta}_\theta$ is a non-increasing scalar function. Together with the fact that $\tilde{\eta}_\theta^\top \tilde{\eta}_\theta$ is bounded from below, we know that $\tilde{\eta}_\theta(t)$ has a limit as $t \to \infty$. By virtue of the monitoring signal (30) and the switching logic (28), this indicates that the switching will ultimately stops. Although, this does not imply that $\eta(t) \to \theta$ or that $\hat{\theta}_{\sigma(\infty)}$ is the one closer to the true value θ.

V. Illustrative Examples

In this section, a simulation study is presented to validate the proposed methodology.

Example 1. Consider a stable non-minimum phase plant model described by

$$\dot{x} = Ax + B[u(t) - d(t)], \quad x(0) = (-1, 1)^\top$$
$$y = Cx$$

where $A = \begin{pmatrix} -0.2 & -0.03 \\ 1 & 0 \end{pmatrix}, B = (1 \quad 0)^\top, C = (2 \quad -2)$. and disturbance signal $d(t) = 2\sin(2t - \frac{\pi}{3})$. Select the controller gain $\varepsilon = 0.5$, the observer gain $\alpha = 0.3$, the adaptive gain $\gamma = -0.005$, and the switching threshold $h = 0.1$. The candidate controller set $\{\mathcal{C}^i\}$ is constructed in a way that the $\hat{\theta}_i$ are evenly distributed in $\varTheta$, as shown in Figure 3. In this example, we have 21 candidate controllers in total.

Figure 3 shows the trajectory of estimator $\eta(t)$ (blue line) and the sequence of active candidate controller (red dashed line with arrow). To show the effectiveness of the proposed scheme, we initialize the controller with the worst case scenario where the sign of the components of $\hat{\theta}_{\sigma(0)} = \eta(0) = (-0.76, -0.76)$ is opposite to the one of the true value $\theta^* = (0.599, 0.947)$. Figure 4 shows the time history of model output y, active controller index σ and model estimator η, respectively. It can be seen that the switching-based AFC successfully rejects the disturbance within 100 [s] with three switches. We observe that the estimator crosses the inner circle and eventually converges to a value that is not equal to the true value, but has the same sign. Accordingly, the switching also stops, as we expected.

Example 2. Next, we consider a more challenging case where an abrupt change of model parameters occurs at

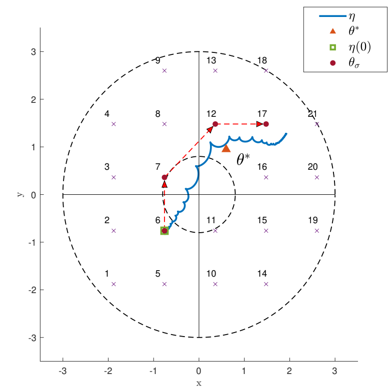

Fig. 3. Trajectory of η, location of θ^* and activated parameter vector θ^j for Example 1.

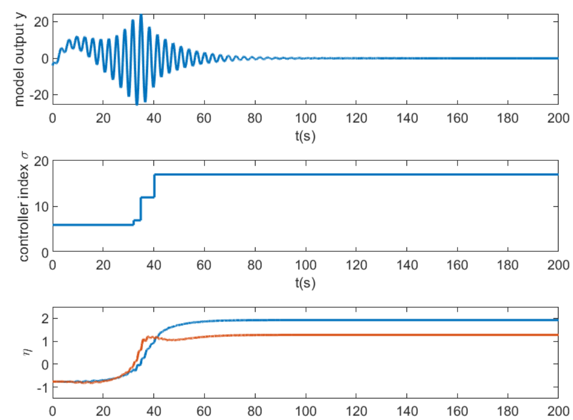

Fig. 4. Time-history of system output y, σ and η for Example 1.

$t = 200[s]$, namely the matrix C changes to $(-2 \quad 2)$ at $t = 200[s]$, which leads to a change in the frequency response from $\theta_a^* = (0.599, 0.947)$ to $\theta_b^* = (-0.599, -0.947)$. The tuning parameters and setting of candidate controllers are kept the same as on Example 1. Figures 5 and 6 show that η approaches θ_a^* first, which is the same as in Example 1. After the change of model has taken place, it takes about 150 [s] for the system to react. Note that this lag depends mainly on the time constants of the plant. When the unsuitability of the current controller is detected, the supervisor is engaged, and $\dot{\eta}$ changes its direction and the estimate converges towards θ_b^*. Finally, after another 200 $[s]$ and four switches, the supervisor selects a stabilizing controller that achieves the rejection of the disturbances.

VI. Concluding Remarks

This paper proposed a novel switching-based AFC to reject periodic disturbances in the absence of SPR-like conditions, which are required by the majority of existing AFC approaches. Boundedness of closed-loop trajectories asymptotic convergence of the output are proved. Furthermore, compared to previous schemes, the transient behaviour is significantly improved without increasing the complexity of the algorithm. Hence, this work lays the basis for solutions

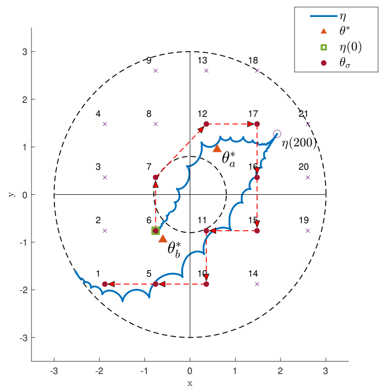

Fig. 5. Trajectory of η, position of θ_a and θ_b for two models and activated parameter vector $\hat{\theta}_{\sigma(t)}$ for Example 2.

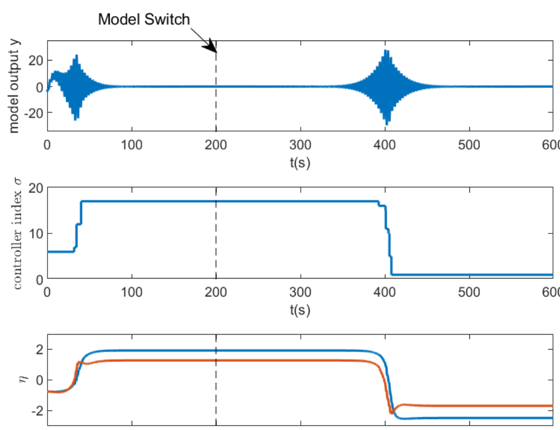

Fig. 6. Time-history of system output y, σ and η for Example 2.

tackling more complicated disturbance rejection problems with multi-sinusoidal signals.

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